January 2006 PHYS393 Examination

1.

(a). A set of 4 distinguishable particles occupies energy states 0, ε, 2ε, 3ε, 4ε ……
The total energy of the set is 5ε.

- (i) Write out the 6 possible distributions **[2]**
- (ii) Evaluate the number of microstates for each distribution **[2]**
- (iii) Evaluate the mean populations of the states. **[2]**

(b) N atoms bound into a solid system at temperature T can each exist in states of energy 0, ε, 2ε.

(i)Write an expression for the Partition function of the atoms. **[2]**

(ii) Using the bridge relation

$$U = NkT^2 \frac{\partial}{\partial T}(\ln Z)$$

or otherwise, show that the internal energy U can be written as

$$U = \frac{N\varepsilon.\exp(-\varepsilon/kT).[1+2\exp(-\varepsilon/kT)]}{[1+\exp(-\varepsilon/kT) + \exp(-2\varepsilon/kT)]}$$ **[2]**

(iii)Derive the limiting values of U as $T \rightarrow 0$ and as $T \rightarrow \infty$. **[2]**

(iv)Sketch a graph of U versus T. **[2]**

(v) Without further differentation sketch the graph of C_V versus T. **[2]**

(c) The Maxwell-Boltzmann distribution of speeds of molecules n(v) in a box containing N molecules of mass m at temperature T is written

$$n(v) = 4\pi N.(m/2\pi kT)^{3/2} v^2 \exp(-mv^2/2kT)$$

(i) Draw the distribution n(v) versus v. **[2]**

(ii) Derive an expression for the most probable velocity v_p. **[2]**

(iii) Derive an expression for the mean square speed $v^2{}_m$ **[2]**

(iv) Estimate the energy of a mole ($6\text{x}10^{23}$ molecules) of a monatomic gas at room temperature (300K). **[2]**

$$\text{Integrals } I_n = \int_o x^n \exp(-bx^2)$$

Can be evaluated from

$$I_n = [(n-1)/2b]\, I_{n-2}\ , \qquad I_0 = \tfrac{1}{2}(\pi/b)^{1/2}\ , \qquad I_1 = 1/2b$$

(d) The molecule HD is composed of an atom of hydrogen (H, atomic mass = 1) bonded to an atom of deuterium (D, atomic mass = 2).
Vibrational energy levels of such molecules are given by

$$E(n) = (n+½)\,\hbar\,(B/\mu)^{½}$$

where B is the inter-atomic force constant, μ is the reduced mass of the molecule and n is an integer.
Rotational energy levels are given by the relation

$$E(I) = \frac{\hbar^2 I(I+1)}{2\mu r^2}$$

where the integer I is the quantised angular momentum and r is the separation of the atoms.
(i) Evaluate excitation energies for the first excited states of vibrational and rotational motion for the HD molecule in which $B = 6.91 \times 10^2$ kg.s^{-1} and $r = 1.05 \times 10^{-10}$m. **[2]**
(ii) Estimate temperatures θ_V and θ_R at which vibrational and rotational motions become populated for the HD molecule. **[2]**
(iii) Give reasoned estimates for the heat capacity C_V of a mole of HD at temperatures T = 20K and at T = 300K. **[2],[2]**

(e)
(i) Draw diagrams depicting the constituents of an atom of He^3 and an atom of He^4. **[2]**
(ii) In each case specify the electronic, nuclear and total angular momentum values of the atom **[2]**
(iii) Explain why one type of atom is a fermion and the other is a boson. **[2]**
(iv) He^3 atoms in liquid form at T = 2K in a container occupy a level scheme of quantised states. Sketch the occupation of these levels. **[2]**
(v) He^4 atoms in liquid form at T = 2K in a container occupy a level scheme of quantised states. Sketch the occupation of these levels. **[2]**

(f) The critical field $B_C(T)$ in a superconductor at temperature T is related to that at T = 0, $B_C(0)$, by the relation

$$B_C(T) = B_C(0).[1-(T/T_C)^2]$$

where T_C is the critical temperature at B = 0.
(i) Sketch the relation $B_C(T)$ versus T. **[2]**
(ii) Label the regions of superconducting and normal behaviour on the sketch **[2]**
(iii) The element lead has $T_C = 7.2$K and $B_C(0) = 0.080$T. Calculate the critical temperature for lead in a field of B = 0.05T. **[2]**
(iv) Is lead at a temperature of T = 5.6K in an applied field of B = 0.035T in a superconducting or a normal state? **[2]**

2. Answer **either** 2(a) **or** 2(b)

2(a) A system is composed of N conduction electrons which move freely within a cube of metal of side L.

(i)Write quantised values for the wavevector components k_x, k_y, k_z corresponding to allowed motions of the electrons. **[2]**

(ii) Illustrate the allowed states in k_x, k_y, k_z space. **[2]**

(iii)Taking account of the spin degeneracy of electrons, show that the number of states g(k) with values of k in the range k to k +dk is

$$g(k)dk = \frac{2.V.4\pi k^2.dk}{(2\pi)^3}$$

where the volume $V = L^3$. **[4]**

(iv) Write the relation between k and energy ε. **[1]**

(v) Show that the number of states with energy in the range ε to ε + dε is

$$g(\varepsilon)d\varepsilon = \frac{2.V.(2m/\hbar^2)^{3/2}.\varepsilon^{1/2}.d\varepsilon}{(2\pi)^2}$$ **[3]**

(vi) The probability that a state with energy ε will be occupied is f (ε).
Sketch graphs of f (ε) versus ε for electrons
For temperature T = 0
For temperature $0 < T < T_F$ where T_F is the Fermi temperature.
Indicate the Fermi energy μ(0) at T = 0. **[3]**

(vii) Show that μ(0) is given by

$$\mu(0) = (\hbar^2 /2m).(3\pi^2N/V)^{2/3}$$ **[4]**

At temperature T the internal energy U of the electrons can be written

$$U = \frac{2N\mu(0)}{3} + \frac{N\pi^2.(kT)^2}{4\mu(0)}$$

Metallic silver has a molar volume of 10.27×10^{-6} m^3 and each atom contributes one electron to the conduction band.

(viii) Evaluate the Fermi energy μ(0) for silver. **[3]**

(ix) Evaluate the electron heat capacity C_V for a molar quantity of silver at a temperature of 5K. **[3]**

2(b)

(i)Describe what is meant by black body radiation. **[4]**

The density of states g(k) in terms of wave-vector k for quantised electromagnetic waves in a cavity of volume V is written as

$$g(k)dk = 2.\frac{V.4\pi k^2\, dk}{(2\pi)^3}$$

(ii) Show that this density of states can be written in terms of the frequency ν of the waves as

$$g(\nu)d\nu = \frac{8\pi V\nu^2\, d\nu}{c^3}$$

where c is the velocity of light. **[3]**

(iii)The energy contained in the frequency interval ν to ν + dν of the radiation is given by

$$\varepsilon(\nu)d\nu = \frac{8\pi V\nu^2 d\nu}{c^3} .\ h\nu .\ \frac{1}{[\exp(h\nu/kT - 1]}$$

Explain the meaning of the factors hν and $\frac{1}{[\exp(h\nu/kT) - 1]}$ in this expression. **[2]**

(iv)Deduce the limiting values of ε(ν) as ν → 0 and as ν → ∞. **[2[,[2]**

(v)Sketch the distribution of ε(ν) versus ν and relate the energy density (U/V) to the sketch. **[4]**

(vi) Using the integral given below – show that

$$(U/V) = \frac{8\pi^5 .\ (kT)^4}{15 .\ (hc)^3}$$

[4]

(vii) Evaluate the energy density at T = 1000K **[2]**

(viii) Calculate the temperature at which the energy density is 10 times greater than it is at T = 1000K. **[2]**

$$\frac{y^3\, dy}{[\exp(y) - 1]} = \frac{\pi^4}{15}$$

Answer **either** 3(a) **or** 3(b).

3(a)
(i)Describe the basic features of the microscopic theory of superconductivity. **[4]**

Use this theory to explain:

(ii) the nature of the current carriers in the normal and superconducting states **[2]**

(iii) the critical temperature T_C and the critical field H_C of a superconductor **[3]**

(iv) the isotope effect **[3]**

(v) the comparison between the resistivity of a non-superconducting metal (Cu) and the normal phase of a superconducting metal (Pb). **[3]**

(vi) the nature of high T_C superconductors **[3]**

(vi) Describe the Meissner effect. **[3]**

(viii) Draw graphs to illustrate the behaviour of Type I and Type II superconductors **[2]**

(ix) Give an example of a Type I and a Type II superconductor. **[2]**

3(b)
Draw the pressure versus temperature phase diagram for He^4 and label the different phases. **[4]**
Describe a model for liquid He II and show how this model explains the measurements viscosity for this liquid phase. **[4]**
Draw the pressure versus temperature phase diagram for He^3 and label the different phases. **[4]**
Does liquid He^3 show superfluid properties? If so give a brief description of how these may be explained. **[2]**
Why may parts of the phase diagram be sensitive to the application of a magnetic field? **[2]**

Describe **one** method of attaining temperatures of about 1mK using liquid helium. The description should include the type of helium used, the basic theory of the method, a schematic sketch of the apparatus and the starting temperature of the process. **[9]**